Title: Speaking Parametrically!

Link to Outcomes:

- **Problem Solving** Students will demonstrate their ability to solve distance, rate, and time problems.
- **Communication** Students will communicate their observations by writing down what they see occurring on the calculator related to the problem being illustrated. The students will communicate orally what they observe.
- **Reasoning** Students will follow steps to set up distance, rate, and time problems. They will analyze solutions and apply techniques to differing types of distance problem.
- **Connections** Students will use equations to solve real-world problems involving distance, rate, and time.
- **Technology** Students will use the TI-82 calculator in the parametric mode to solve distance, rate, and time problems.
- **Algebra** Students will use algebra techniques to determine equations that will allow them to solve the problems.

Brief Overview:

Students will use the TI-82 graphing calculator in parametric mode to solve distance, rate, and time problems.

Initial activity will be teacher directed to show students how to use the calculator in parametric mode. The students will work along with the teacher to solve a problem . The students will be required to write their observations of what is occurring on the calculator and explain what it means with respect to the given problem.

The class will be separated into cooperative learning groups to set up simulations of five different problems. They must submit a written report describing their findings on each of the five problems.

Grade/Level:

Grades 7 – 12; Pre-algebra, Algebra I, Algebra II
(Note: This lesson can be adapted for higher-level classes by changing the problems assigned. We have included a couple of these problems.)

Duration/Length:

This unit will take two to three 45-minute periods or two 90-minute classes.

Prerequisite Knowledge:

Students should know basic use of the TI-82 calculator. They should know basics of setting up equations.

Objectives:

- Use a TI-82 calculator in parametric mode
- Analyze the results of inputting information into the calculator.
- Be able to set up the correct parameters to solve different problems involving distance, rate, and time.

Materials/Resources/Printed Materials:

- TI-82 calculators
- Student Notes #1
- Student Worksheet #1
- Student Worksheet #2
- Student assessment #1
- Student assessment #2
- Enrichment/advanced worksheet "Going, Going, and Gone?"
- Teacher Resource Sheet for forming cooperative learning groups

Development/Procedures:

Teacher will demonstrate the method for changing the TI-82 calculator into parametric mode. The students will follow along with the teacher putting their calculator into the proper format. Teacher and students will work together on the first problem. The students will follow along with student notes #1.

The class will then be formed into teams using the cards with equations the students must solve. The students search the room for other students in the class with equivalent equations. This team of students will form the team of 4.

Each group will first complete student worksheet #1. A set of problems to solve using their TI-82 will be given to each group for completion. Each group will be responsible for presenting the results to the class using the calculator, diagrams and oral description of their findings.

The students will be assigned problems to be completed individually for assessment.

Evaluation:

The students will be evaluated based on their presentation to the class as a group. They will also be evaluated on their individual problem for which they must submit a written report.

Extension/Follow Up:

Students will be given more challenging problems to solve and/or will be asked to design their own problems to be presented to other members of the class.

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Student Notes #1: Solving Problems Using Parametric Form

Directions to change TI-82 to Parametric Mode

- Press {MODE}
- Using the down arrow move the cursor to the 4th line.
- Using the arrow keys move to the right {PAR}, Press {ENTER}.
- Using the down arrow move the cursor to the 6th line.
- Using the arrow keys to highlight **simul**, Press {ENTER}.

Consider the Problem:

Amy leaves Baltimore, MD by train traveling at 70 mph heading towards Dallas, Texas. Her fiancee Doug leaves 12 hours later by plane traveling at a speed of 500 mph. The distance from Baltimore to Dallas is 1250 miles. When will Doug and Amy be the same distance from Baltimore?

Recall that d = r•t (distance = rate x time)

- Write an expression which represents the distance Amy has traveled in T hours._____
- Write an expression which represents the distance Doug has traveled in (T-12) hours._____
- Press {Y=}
  - To clear out other expressions, press {CLEAR} then {ENTER} after each parametric variable.
  - You will see the following:

x_{1T} = ; you type 70 Press {X,T,θ} to get T down arrow

y_{1T} = ; you type 3

x_{2T} = you type 500(T-12)

y_{2T} = you type 6

- Press {WINDOW}
  Set up your window as follows:

$T_{min} = 0$

$T_{max} = 24$

$T_{step} = 1$

$X_{min} = 0$

$X_{max} = 1300$

$X_{scl} = 130$

$Y_{min} = 0$

$Y_{max} = 10$

$Y_{scl} = 1$

To put these values into the window move the cursor down using the down arrow. Type over the numbers that are there and press {ENTER}.

- Press {GRAPH} ; Observe the graph. What happens?

- To slow down the graphing process go back to {WINDOW} and change the T_{step} to .5 and press {GRAPH}

- We want to stop Doug when he is directly over Amy. We must regraph the problem. Again we want to change the T_{step}. Change it to 0.1 this time to slow it down more. To stop the movement of the graphs on the screen press {ENTER} after you have pressed {GRAPH}. Use a **non-permanent** marking pen to mark where Doug passes over Amy.

Note: To re-graph you must go into the {WINDOW} and change the T_{step} value.

- To identify the interval in which the plane overtakes the train we will use the trace feature. Press {TRACE}. Move the cursor (using the right arrow) on the path of the train to the spot at which you think the plane overtook it. Observe the values for x and T on the bottom of your screen. Record these values below in the table. Use the up arrow to move to the path of the plane and then the right arrow or left arrow to move back and forth on this line. Again observe the values of x and T and record the values in the table below. Using the table determine the interval in which the plane overtook the train.

Amy		**Doug**
x1	**T**	**x2**

Teacher Notes/Answer Key

Student Notes #1; Solving Problems Using Parametric Form

Directions to change TI-82 to Parametric Mode

- Press {MODE}
- Using the down arrow move the cursor to the 4th line.
- Using the arrow keys move to the right {PAR}, Press {ENTER}.
- Using the down arrow move the cursor to the 6th line.
- Using the arrow keys to highlight **simul**, Press {ENTER}.

Consider the Problem:

Amy leaves Baltimore, MD by train traveling at 70 mph heading towards Dallas, Texas. Her fiancee Doug leaves 12 hours later by plane traveling at a speed of 500 mph. The distance from Baltimore to Dallas is 1250 miles. When will Doug and Amy be the same distance from Baltimore?

Recall that d = rt (distance = rate x time)

- Write an expression which represents the distance Amy has traveled in T hours.
  d=70t
- Write an expression which represents the distance Doug has traveled in (T-12) hours.
  d=500(t-12)
- Press {Y=}
  - To clear out other expressions, press {CLEAR} then {ENTER} after each parametric variable.
  - You will see the following:

x_{1T} = ; you type 70 Press {X,T,θ} to get T down arrow
y_{1T} = ; you type 3
x_{2T} = you type 500(T-12)
y_{2T} = you type 6

- Press {WINDOW}
  Set up your window as follows:

$T_{min} = 0$
$T_{max} = 24$
$T_{step} = 1$
$X_{min} = 0$
$X_{max} = 1300$
$X_{scl} = 130$
$Y_{min} = 0$
$Y_{max} = 10$
$Y_{scl} = 1$

To put these values into the window move the cursor down using the down arrow. Type over the numbers that are there and press {ENTER}.

- Press {GRAPH} ; Observe the graph. What happens?

- To slow down the graphing process go back to {WINDOW} and change the T_{step} to .5 and press {GRAPH}

- We want to stop Doug when he is directly over Amy. We must regraph the problem. Again we want to change the T_{step}. Change it to .1 this time to slow it down more. To stop the movement of the graphs on the screen press {ENTER} after you have pressed {GRAPH}. Use a **non-permanent** marking pen to mark where Doug passes over Amy.

Note: To re-graph you must go into the {WINDOW} and change the T_{step} value.

- To identify the interval in which the plane overtakes the train we will use the trace feature. Press {TRACE}. Move the cursor (using the right arrow) on the path of the train to the spot at which you think the plane overtook it. Observe the values for x and T on the bottom of your screen. Record these values below in the table. Use the up arrow to move to the path of the plane and then the right arrow or left arrow to move back and forth on this line. Again observe the values of x and T and record the values in the table below. Using the table determine the interval in which the plane overtook the train.

Amy		Doug
x1	T	x2
994	14.2	1100
980	14	1000
966	13.8	900
973	13.9	950
976.5	13.95	975

By this table it can be determined that the time interval in which Doug passed Amy was greater than 13.95 and less than 14 hours after Amy left and greater than 1.95 hours and less than 2 hours for Doug.

Possible answers for the table are given. They will vary depending on the Tstep used.

Student Worksheet #1

1. Write a paragraph describing what you observed on the calculator and explain what X_{1T}, X_{2T}, Y_{1T}, Y_{2T}, and T mean in this situation. What does the T_{step} mean?

2. Experiment with your equations to determine when Doug should leave to arrive approximately the same time as Amy in Dallas. Write the best equation you find below.

3. Suppose Amy and Doug were starting from opposite directions. Let's suppose Amy is in Dallas and Doug is in Baltimore. They decide to surprise each other with a visit. If Amy still takes the train (she doesn't like to fly) traveling the same speed and Doug leaves 12 hours later on a plane going 500 mph.

   a. X_{1T} = _____ X_{2T} = _____
   Y_{1T} = _____ Y_{2T} = _____

b. Determine the approximate time Doug and Amy will cross paths.

c. Write a paragraph describing what you observed on the calculator and explain what it means.

Teacher Notes/Answer Key

Student Worksheet #1

1. Write a paragraph describing what you observed on the calculator and explain what X_{1T}, X_{2T}, Y_{1T}, Y_{2T}, and T mean in this situation. What does the T_{step} mean?

   Answers will vary: The students should describe the motion they see on the calculator and explain what it means. The x values represent distance in miles, the y just represents the path used on this coordinate plane, and T is the time in hours. The T_{step} changes the time interval, and therefore the smaller the T_{step} the slower the graph will move.

2. Experiment with your equations to determine when Doug should leave to arrive approximately the same time as Amy in Dallas. Write the best equation you find below.

   Answers will vary: Based on the answers found in the table with the teacher directed activity students should be able to come up with an equation which would model them arriving at the same time. The equation for Doug might be $x_{2t} = 500(t-15)$

3. Suppose Amy and Doug were starting from opposite directions. Let's suppose Amy is in Dallas and Doug is in Baltimore. They decide to surprise each other with a visit. If Amy still takes the train (she doesn't like to fly) traveling the same speed and Doug leaves 12 hours later on a plane going 500 mph.
   •Set up a model in parametric mode which will illustrate their travel and determine when their paths cross.
   [Remember they are starting at different places]
   a. X_{1T} = **70T** X_{2T} = **-500(t-12)+1250**

   Y_{1T} = **3** Y_{2T} = **6**

   b. Determine the approximate time Doug and Amy will cross paths.
   Answers will vary: In approximately 15 hours 24 minutes they cross paths..

   c. Write a paragraph describing what you observed on the calculator and explain what it means.
   Answers will vary: The students should observe that the two are moving towards each other in opposite directions.

Forming Cooperative Learning Groups

1. Teacher Preparation:

   Make up an index card for each student in the class using sets of linear equations in one variable. The teams can be completely random allowing students to pick their own card or you can specify a student's name on the card to result in teacher generated groups.

2. Group Formation:

   Hand out cards to the students. The students must first solve the equations. After finding the solutions they must circulate around the room to find 3 other students with the same solution.

Group 1	$24 = 5y + y$ $5a - 3\frac{1}{4}a = 7$	$7 + 5y = 27$ $12x - 6 = 8x + 10$
Group 2	$6x + 3x = 18$ $5y + (-4y) = 2$	$4x - 6 = 3x - 4$ $2(3x + 4) = 20$
Group 3	$y - 6y = -30$ $\frac{1}{3}y + \frac{5}{3}y = 12$	$m + m = 12$ $9 - (2a + 6) = -9$
Group 4	$-7y = -35$ $x + 4x = 25$	$9x - 6x = 15$ $7y - 6 = 4y + 9$
Group 5	$s + \frac{1}{8} = \frac{5}{8}$ $7 - n = 5 + 3n$	$v - \frac{1}{6} = \frac{1}{3}$ $8z - 6 + 7z = 11z - 4$
Group 6	$7 = 4 - y$ $-24 = 8m$	$8x - 9 = x - 30$ $5z + 3z - 6 = 6(z - 2)$
Group 7	$13x = 91$ $3y + 2 = 23$	$7x - 9 = 3y + 19$ $8b + 3(1 - b) = 2(3b - 2)$
Group 8	$5x + 7x = 12$ $a - 4 = -3$	$3(x + 2) = x + 8$ $\frac{2}{3}a - \frac{1}{6} = \frac{1}{2}a$

Student Worksheet #2

Directions: Solve the following problems using the TI-82 calculator in parametric mode. Be prepared to explain your findings to the class. Your group will be asked to share your equations, with appropriate diagrams, and to use the TI-82 overhead to demonstrate the problem.

1) Irv started driving to Ocean City at 40 mph. Two hours later Jerilyn left from the same point. She drove along the same road at 50 mph. How many hours had she driven before she caught up with Irv?

2) Sean was racing Gina. Sean runs an average of 20 feet per second, while Gina runs an average of 15 feet per second. Sean gives Gina a 3 second head start. In how many seconds will Sean catch up with Gina.

3) A USAir plane leaves BWI airport flying at a rate of 500 mph heading for Puerto Rico. An American Airlines plane leaves one hour later from BWI traveling at 600 mph for the same destination. When will the two planes be the same distance from the airport?

4) Two trucks started towards each other at the same time from towns 315 miles apart. One truck averaged 55 mph and the other averaged 50 mph. After how many hours did they pass each other?

5) Alisa and Shawnetta agreed to meet at the mall. Alisa lives due west of the mall and 20 miles from Shawnetta who lives due east of the mall. Alisa leaves home one-fifth of an hour after Shawnetta. Her average rate is 40 mph while Shawnetta can only travel at 30 mph. How long before each arrives to the mall.

Teacher Notes/Answer Key
Student Worksheet #2

Directions: Solve the following problems using the TI-82 calculator in parametric mode. Be prepared to explain your findings to the class. Your group will be asked to share your equations, with appropriate diagrams, and to use the TI-82 overhead to demonstrate the problem.

1) Irv started driving to Ocean City at 40 mph. Two hours later Jerilyn left from the same point. She drove along the same road at 50 mph. How many hours had she driven before she caught up with Irv?

Window
Tmin = 0
Tmax =6
Tstep = 1
Xmin = 0
Xmax = 400
Xscl = 40
Ymin = 0
Ymax = 10
Yscl = 1

Parametric Equations
$x_{1T} = 40t$
$y_{1T} = 3$
$x_{2T} = 50(t-2)$
$y_{2T} = 6$

She had driven 8 hours before she caught up to Irv. This only happens if Ocean city is 400 miles from their home. If they were leaving from Baltimore he could not overtake her.

2) Sean was racing Gina. Sean runs an average of 20 feet per second, while Gina runs an average of 15 feet per second. Sean gives Gina a 3 second head start. In how many seconds will Sean catch up with Gina.

Window
Tmin = 0
Tmax =20
Tstep = 1
Xmin = 0
Xmax = 500
Xscl = 50
Ymin = 0
Ymax = 10
Yscl = 1

Parametric Equations
$x_{1T} = 15t$
$y_{1T} = 3$
$x_{2T} = 20(t-3)$
$y_{2T} = 6$

After she has been running for 12 seconds Sean will catch up. Sean will catch up in 9 seconds after he starts running.

3) A USAir plane leaves BWI airport flying at a rate of 500 mph heading for Puerto Rico. An American Airlines plane leaves one hour later from BWI traveling at 600 mph for the same destination. When will the two planes be the same distance from the airport?

Window
Tmin = 0
Tmax =20
Tstep = 1
Xmin = 0
Xmax = 3500
Xscl = 100
Ymin = 0
Ymax = 10
Yscl = 1

Parametric Equations
$x_{1T} = 500t$
$y_{1T} = 3$
$x_{2T} = 600(t-1)$
$y_{2T} = 6$

The planes will both be 3000 miles from the airport 6 hours after the US Air plane leaves BWI.

Teacher Notes/Answer Key
Student Worksheet #2

4) Two trucks started towards each other at the same time from towns 315 miles apart. One truck averaged 55 mph and the other averaged 50 mph. After how many hours did they pass each other?

Window
Tmin = 0
Tmax =20
Tstep = .05
Xmin = 0
Xmax = 315
Xscl = 30
Ymin = 0
Ymax = 10
Yscl = 1

Parametric Equations
$x_{1T} = 55t$
$y_{1T} = 3$
$x_{2T} = -50t+315$
$y_{2T} = 6$

The trucks will pass each other 3 hours after they leave.

5) Alisa and Shawnetta agreed to meet at the mall. Alisa lives due west of the mall and 20 miles from Shawnetta who lives due east of the mall. Alisa leaves home one-fifth of an hour after Shawnetta. Her average rate is 40 mph while Shawnetta can only travel at 30 mph. How long before each arrives at the mall?

Window
Tmin = 0
Tmax =20
Tstep = .01
Xmin = 0
Xmax = 25
Xscl = 5
Ymin = 0
Ymax = 10
Yscl = 1

Parametric Equations
$x_{1T} = 30t$
$y_{1T} = 3$
$x_{2T} = -40(t-1/5)+20$
$y_{2T} = 6$

Shawnetta arrives in 24 minutes and Alisa arrives at the same place 12 minutes after she leaves.

Assessment #1

Use the methods discussed in class to complete the following problem.

An Amtrak train left Union Station traveling 125 mph heading for Disney World in Florida. Another train left 2 hours later on a different track heading for the same location at 175 mph. The distance is approximately 1225 miles.

1. Did the second train overtake the first before they arrived at Disney World? If so after how many hours did the second train overtake the first train?

2. Write a paragraph explaining the procedure you used to solve this problem.

Assessment #2

Use the methods discussed in class to complete the following problem.

Michael and Lisa Marie are vacationing in Jamaica. Their cruise ship leaves Miami with Lisa Marie, but Michael was left snorkeling.

Michael surfaces, panics, showers, and eats dinner. He then contacts the island heliport to check on helicopter rates, which are $150.00 per hour. Michael decides to contract the helicopter to take him to the ship where he will parachute to Lisa Marie.

He leaves Jamaica 3 hours after the ship left. It is 550 miles from Jamaica to Miami. The ship travels at 30 mph, the helicopter at 120 mph. When he calls Western Union for $$$, how much does he need?
Remember: if he misses the boat, he's shark bait!!

P.S. The wind is not blowing.

Extension/Follow-up Activity for More Advanced classes

GOING, GOING, GONE?

It's the bottom of the ninth, two outs, and the Orioles are behind, 6-3.

However, the bases are loaded and Cal Ripkin is at the plate. He swings – it looks like a hit! His bat makes contact with the ball three feet above the ground at an angle of 20 degrees and a velocity (bat speed) of 150 feet per second. He hits straight toward center field where there is a fence 20 feet high and 400 feet from home plate. At the moment the ball is hit, there is a 6 mph wind blowing straight in from center field.

Does Ripken hit a home run? If not, are the Orioles still in the inning? Would the outcome change any if there were no wind?

Teacher Notes/Answer Key

Assessment #1

Window
Tmin = 0
Tmax =10
Tstep = 1
Xmin = 0
Xmax = 1000
Xscl = 100
Ymin = 0
Ymax = 10
Yscl = 1

Parametric Equations
$x_{1T} = 125t$
$y_{1T} = 3$
$x_{2T} = 175(t-2)$
$y_{2T} = 6$

1. The second train did overtake the first train before they reached Disney World. It took 5 hours after the second train left and 7 hours after the first train left.

2. The students should be able to describe the method they used on the calculator etc.

Assessment #2

Window
Tmin = 0
Tmax =20
Tstep = .1
Xmin = 0
Xmax = 550
Xscl = 50
Ymin = 0
Ymax = 10
Yscl = 1

Parametric Equations
$x_{1T} = 30T$
$y_{1T} = 3$
$x_{2T} = 120(T-3)$
$y_{2T} = 6$

It would take him 4 hours to reach the ship and it would cost him $600.